

A systematic method for converting a polar equation to rectangular form that can be used in many cases

The following method might not be the most efficient for every problem, but it will almost always get the correct answer.

eg.
$$r = \frac{1}{1 + \sin 2\theta}$$

- [1] Use identities to replace all trigonometric functions involving sums/differences of angles, 2θ , $\frac{1}{2}\theta$ etc. with equivalent functions using only $\sin \theta$, $\cos \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, or $\cot \theta$.

$$r = \frac{1}{1 + 2 \sin \theta \cos \theta}$$

- [2] Multiply & simplify the equation to eliminate all fractions in which the denominator contains r and/or θ .

$$r(1 + 2 \sin \theta \cos \theta) = 1$$

- [3] Replace all trigonometric functions of θ with their equivalents in x , y and r .

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x} \quad \csc \theta = \frac{r}{y} \quad \sec \theta = \frac{r}{x} \quad \cot \theta = \frac{x}{y}$$

$$r \left(1 + 2 \frac{y}{r} \frac{x}{r} \right) = 1$$

- [4] Multiply & simplify the new equation to eliminate all fractions in which the denominator contains r and/or x and/or y .

$$r + \frac{2xy}{r} = 1$$

$$r^2 + 2xy = r$$

- [5] Replace r with $(x^2 + y^2)^{\frac{1}{2}}$. Simplify exponents. Square (if necessary) to eliminate fractional exponents. Expand (if there will be like terms) and collect like terms.

$$((x^2 + y^2)^{\frac{1}{2}})^2 + 2xy = (x^2 + y^2)^{\frac{1}{2}}$$

$$x^2 + y^2 + 2xy = (x^2 + y^2)^{\frac{1}{2}}$$

$$(x^2 + y^2 + 2xy)^2 = ((x^2 + y^2)^{\frac{1}{2}})^2$$

$$((x + y)^2)^2 = x^2 + y^2$$

$$(x + y)^4 = x^2 + y^2$$

NO LIKE TERMS WILL APPEAR, SO NO EXPANSION